Consider the curve defined by

$$x = t - 3\sin(t)\,;\quad y = 4 - 3\cos(t)$$

$$0 \leq t \leq 10 \ \ldots$$

a) can you graph this?

b) is this a function?

c) when $t=0$ where are we?

$x(0) = 0 - 3\sin 0 = 0$ $\qquad$ $y = 4 - 3(\cos(0)) = 4 - 3 = 1$

d) how do I find $\frac{dy}{dx}$?

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \quad \left[\text{as long as } \frac{dx}{dt} \neq 0\right]$$

$y(t) = 4 - 3\cos(t)$ $\qquad$ $x(t) = t - 3\sin(t)$

$\frac{dy}{dt} = y'(t) = +3\sin(t)$ $\qquad$ $\frac{dx}{dt} = x'(t) = 1 - 3\cos(t)$

$$\frac{dy}{dx} = \frac{3\sin t}{1 - 3\cos(t)} \quad (1 - 3\cos t \neq 0)$$

e) $$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{(3\cos t)(1-3\cos t) - (3\sin t)(3\sin t)}{(1-3\cos t)^2}}{1 - 3\cos t}$$

i.e. complicated

another example

$x = \cos(t) \quad y = \sin(t)$

→ graph   circle… what does t need to be?

→ what is the shape of the path.

→ find $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos t}{-\sin t} = -\cot(t)$

→ find $\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}(-\cot(t))}{\frac{dx}{dt}}$

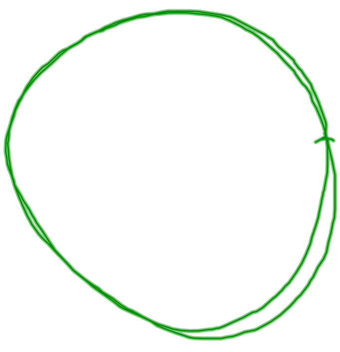
$= \frac{-(-\csc^2 t)}{-\sin(t)} = \frac{\csc^2 t}{-\sin t}$

another way to find $\frac{dy}{dx}$ . . . . .

"eliminate the parameter"

$\cos^2 t + \sin^2 t = 1$

$x^2 + y^2 = 1$



Consider the curve defined by:

$x(t) = t - 3\sin(t)$; $y(t) = 4 - 3\cos(t)$; $0 \le t \le 10$

☆ graph this . . .

☆ is this a function?

☆ when $t = 0$ where are we?

$x(0) = 0 - 3\sin(0) = 0$ $\quad$ $y(0) = 4 - 3\cos(0) = 4 - 3 = 1$

when $t = \pi$ where are we?

$x(\pi) = \pi - 3\sin(\pi) = \pi - 0 = \pi$; $y(\pi) = 4 - 3\cos(\pi) = 4 - 3(-1) = 7$

☆ what is $\frac{dy}{dx}$?

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

☆ $\left[\frac{dx}{dt} \neq 0\right]$

$$\frac{dy}{dt} = \frac{d}{dt}(4 - 3\cos t) = 3\sin t$$

$$\frac{dx}{dt} = \frac{d}{dt}(t - 3\sin(t)) = 1 - 3\cos(t)$$

$$\frac{dy}{dx} = \frac{3\sin(t)}{1 - 3\cos(t)}$$

☆ what is $\frac{d^2y}{dx^2}$?

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

$$= \frac{\frac{d}{dt}\left(\frac{3\sin t}{1 - 3\cos t}\right)}{1 - 3\cos t} = \frac{\frac{(3\cos t)(1 - 3\cos t) - (3\sin t)(3\sin}{(1 - 3\cos t)^2}}{1 - 3\cos t}$$

for you:

$x = \cos(t)\ ;\ y = \sin(t);\ \pi \le t \le \pi$

→ graph

→ what is the shape of the path?

circle

→ find $\frac{dy}{dx}$

$\frac{dy}{dt} = \cos t \quad \Big\| \quad \frac{dx}{dt} = -\sin t \quad \Big\| \quad \frac{dy}{dx} = \frac{\cos t}{-\sin t}$

$= -\cot(t)$

→ find $\frac{d^2y}{dx^2} = \dfrac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \dfrac{\csc^2(t)}{-\sin(t)}$

$\frac{(-\sin t)(-\sin t)-(\cos t)(\cos t)}{\sin^2 t}$

$= \frac{\sin^2 t + \cos^2 t}{\sin^2 t}$

another $\frac{dy}{dx}$

eliminate the parameter

$\sin^2 t + \cos^2 t = 1$

$y^2 + x^2 = 1$

11.2/1) a) Find the slope of the tangent line to the parametric curve

$$x = t^2 + 1 \ ; \ y = \frac{t}{2} \quad @ \ t = -1 \ ; \ t = 1$$

without elim - - - -

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{1}{2}}{2t} = \frac{1}{4t}$$

$$\left.\frac{dy}{dx}\right|_{t=-1} = \frac{1}{4(-1)} = -\frac{1}{4}$$

$$\left.\frac{dy}{dx}\right|_{t=1} = \frac{1}{4(1)} = \frac{1}{4}$$

elim the parameter

$$x = t^2 + 1 \ ; \ y = \frac{t}{2}$$

$$2y = t$$

$$x = (2y)^2 + 1 \Rightarrow X = 4y^2 + 1$$

$\frac{dy}{dx}$:

$$1 = 8y\frac{dy}{dx}$$

$$\frac{1}{8y} = \frac{dy}{dx}$$

when $t = -1$

$$y = \frac{t}{2} = -\frac{1}{2}$$

$$x = t^2 + 1 = (-1)^2 + 1 = 2$$

$$\left.\frac{dy}{dx}\right|_{t=-1} = \frac{1}{8\left(-\frac{1}{2}\right)} = -$$